On the conductivity of lateral-surface superlattices in magnetic fields

1993 J. Phys.: Condens. Matter 5 6991

(http://iopscience.iop.org/0953-8984/5/37/016)

J. Phys.: Condens. Matter 5 (1993) 6991–6998. Printed in the UK

On the conductivity of lateral-surface superlattices in magnetic fields

R B S Oakeshott and A MacKinnon
Blackett Laboratory, Imperial College, London SW7 2BZ, UK

Received 1 February 1993

Abstract. We present results for the band conductivity and the density of states of electrons in realistic two-dimensional periodic potentials and magnetic fields. We show the effects of the number of flux quanta per unit cell in producing Hofstader's-butterfly-like splittings in the limit of small field, but outside the simple tight-binding limit, and for high fields where the dominant effect is the local splitting of Landau levels to form weakly coupled edge states round the minima and maxima of the potential. The broadening and splitting of these states to form Hofstader's butterfly then occurs on a much finer energy scale.

1. Introduction

Experiments on lateral-surface superlattices [1–10] approach the point where effects due to interactions between the period of the lattice, the Fermi wavelength, and the magnetic length should be apparent.

So far experiments on length scales of around 300 nm have shown mostly semi-classical effects [11] from the interaction of the periods of the cyclotron radius and the superlattice potential with both one-dimensional potentials [12–16], and two-dimensional potentials [5, 8–10] although some effects have required the effect of the density of states on the scattering time to be taken into account [17–19]. For smaller systems it is hoped that purely quantum mechanical effects, seen successfully in single systems such as quantum point contacts [20, 21] should be apparent.

Without a magnetic field, the periodic potential should create gaps at the band edges with a reduction in the conductivity. A magnetic field is expected to introduce further gaps depending on the number of flux quanta per unit cell. In simple cases such as a weak potential [22–24], a tight-binding band [22, 25], or hopping between edge states in different unit cells [26, 27] the magnetic field produces a recursively split spectrum.

Real experiments however are not done with an ideal, tight-binding band, but with many electrons in each unit cell, and so many overlapping bands; with inelastic scattering; with a random potential from impurities; ideally with the periodic potential and the cyclotron energy of similar magnitudes to maximize any effects; and with the magnitude of hopping terms in a tight-binding picture depending on the magnetic field and Fermi energy. We therefore present results here for the band conductivity and density of states of electrons in two-dimensional periodic potentials for realistic Fermi energies and magnetic fields. We focus in this paper on the bulk effects which are likely to show the effects of band gaps in a simpler form than situations where edges are important [26]. In practical experiments, the effects of edge states may or may not be important, depending on the geometry of the sample, and the measurement being made.

0953-8984/93/376991+08$07.50

Here, we present results in the low field limit showing the size of the commensurability effects outside the well known tight-binding limit of Hofstader's butterfly, where we expect other physics to dominate [28]. The high-field limit has been considered by others [6], and we describe how the experimentally observable effects in this limit can be understood in a local picture.

In section 2 we describe the model used for the conductivity, and describe the parameters used. In section 3 we show that the method describes the semi-classical cyclotron radius commensurability oscillations, and that the scattering conductivity can be small relative to the band conductance. In section 4 we show the effect of the commensurability between the area per flux quanta, and the area of a unit cell of the periodic potential. At low Fermi energies the band conductivity is suppressed for fractional numbers of flux quanta per unit cell. At large energies and magnetic fields the Landau levels are split into sublevels. A local model describes the basic splitting of the Landau levels. Most of the states can only tunnel weakly between different unit cells, so that the fine structure caused by commensurability effects is hard to observe. Real experiments are commonly performed by varying the magnetic field, keeping gate voltages constant, so we present results for the density of states calculated for fixed Fermi energy, and varying magnetic field.

2. Model

With the assumptions of a fixed scattering time and periodic potential, we calculate the longitudinal band conductivity as done recently by Degani and Leburton [7]:

$$\sigma_{yy} = \frac{e^2\tau}{A}\sum_{k_x}\sum_{k_y}\sum_{n} v_y^2\left(-\frac{\partial f}{\partial E}\right) \tag{1}$$

where τ is the transport scattering time, v_x is the group velocity of the mode in the x direction, A is the area of the system, and f is the Fermi–Dirac distribution function. For simplicity we ignore spin splitting throughout this paper. The validity and the method of solving equation (1) have been discussed in a preceding paper [28].

In the following we use a transport relaxation time $\tau = 38$ ps, corresponding to a mobility of 100 $m^2\ V^{-1}\ s^{-1}$. The quantum relaxation time — the time for an electron to be scattered out of a plane wave state — is typically about a tenth of the transport relaxation time because of the long range nature of the Coulomb potential [29, 30], and we use a value of $\tau_q = 3.8$ ps in the density of states calculations [31].

Since the amplitudes of short-wavelength Fourier components of the potential are small, because of the distance from the gates to the two-dimensional electron gas (2DEG) [32], we use the simple potential

$$V(x, y) = \tfrac{1}{4}V_0\left[\cos(2\pi x/a) + \cos(2\pi y/a)\right]. \tag{2}$$

The peak–peak potential V_0 can be up to about 20 meV and of the order of the Fermi energy [33], and lattice periods down to of the order of $a = 50$ nm are practical. We show here results for $V_0 = 5$ meV and $a = 100$ nm.

3. Low field behaviour

Figure 1 shows the band conductivity, and the density of states for a magnetic field of 0.21 T, as a function of the cyclotron radius. (The cyclotron radius is defined as the cyclotron radius in the absence of the periodic potential, $r_c = m^* v_F/eB$.)

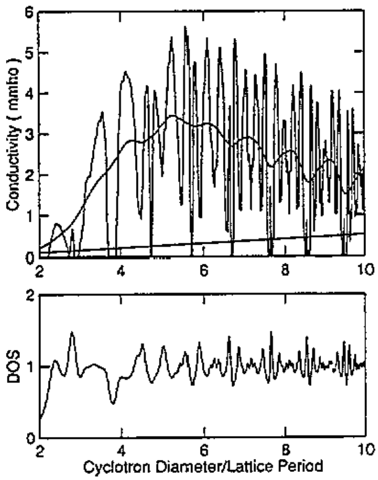

Figure 1. Band conductivity and density of states for a magnetic ield of 0.21 T, $p/q = \frac{1}{2}$. The conductivity is shown at $T = 0$ K and $T = 3$ K. also shown (straight line) is the classical scattering conductivity, multiplied by 25 for clarity. (The cyclotron radius is defined as $r_c = m^* v_F/eB$ where $v_F = \sqrt{2m^* E_F}$.)

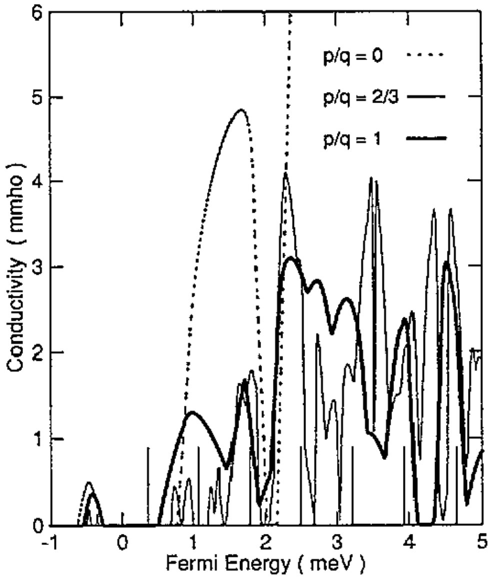

Figure 2. Band conductivity for magnetic fields of 0, 0.28, and 0.41 T. The vertical lines mark the positions of the Landau levels for $B = 0.41$ T.

The overall shape of the band conductivity versus energy graph is determined by two limits. Firstly, the conductivity must vanish as the Fermi energy falls below the minimum of the potential. Secondly, for large Fermi energies, where the cyclotron radius is larger than the lattice period, the dispersion of the Landau levels is smaller, because the electron sees the potential averaged over many unit cells [11]. The conductivity should therefore fall off again as the Fermi energy rises. Středa and MacDonald [34] have described the same behaviour in terms of the probability of magnetic breakdown p [34–37]:

$$p = \exp\left(-\pi V_g^2/4\hbar\omega_c E_F \sin 2\theta\right) \tag{3}$$

where V_g is the size of the band gap at the Brillouin zone edge at zero magnetic field, and 2θ is the scattering angle under Bragg reflection [34].

3.1. Cyclotron oscillations

We now consider some of the fine structure seen in the results. The first thing which the calculations show clearly are the cyclotron radius commensurability oscillations, which have been observed by Weiss and co-workers [9] and others [4, 5, 8, 10].

The phase of the oscillations in the conductivity observed experimentally will depend on whether the band conductivity or the scattering conductivity is dominant. While in experiments with larger lattice periods than the 100 nm considered in this paper the scattering conductivity was dominant [12, 19], as the lattice period is reduced the band conductivity becomes more important.

Figure 1 shows the classical scattering conductivity $\sigma = \sigma_0/(1 + \omega_c^2\tau^2)$ together with the band conductivity. The scattering conductivity for the lattice period considered is small compared with the band conductance. Note that this comparison is sensitive to the magnitude of the periodic potential used, and to the scattering time chosen. To a first approximation, the band conductivity is proportional to the magnitude of the potential, so that for a weaker potential the scattering conductivity will be more important. In addition, the band conductivity is proportional to the scattering time, whereas for $\omega_c\tau \gg 1$ the scattering conductivity varies as $1/\tau$. For large magnetic fields the scattering conductivity will be more important than suggested by this comparison, since the inelastic scattering time will be much smaller when the Fermi energy lies within a Landau level because of the enhanced density of states, while the band conductivity will be reduced because the periodic potential will be strongly screened [38, 39].

4. Quantum commensurability effects

Part of the interest in antidot lattices arises from Hofstader's suggestion [25] that the recursively split band structure arising from the interaction between the lattice periodicity and the magnetic field might be observable in artificial lattices. In a previous paper [28] we have shown results for the splitting in the lowest tight-binding band of the periodic potential. Because of the effects of disorder and electron–electron interactions, the splitting is not expected to be observable in that regime [28]. We now present results for the commensurability effects in two regimes, where the electron density is higher, so that electron–electron interactions and disorder are likely to be less important. The first is the low-field regime with of the order of one flux quantum per unit cell. The second is the high-field regime, where with p/q flux quanta per unit cell, Landau levels are expected to be split into q sub-levels [22].

4.1. Low field commensurability effects

We show now the effects of the magnetic field on the band structure in the low-field regime. Figure 2 shows the band conductivity for zero magnetic field, together with the conductivity for $p/q = \frac{2}{3}$ and $p/q = 1$. ($p/q = 1$ is a magnetic field of 0.4 T.)

At zero magnetic field two separated bands are seen. The lowest is a simple tight-binding band, as analysed by Hofstader [25], and for which we have presented numerical results in a previous paper [28]. The second band is formed in a tight-binding picture from the two p-like states in the dot, and is separated by a gap of about 0.1 meV from the next band. The simple Hofstader splitting is not expected to apply to this band because it is formed from two states, and is not well separated from the other bands.

For $p/q = \frac{2}{3}$ the recursive splitting in the lowest band is clearly resolved, and the conductivity can be seen to be suppressed relative to the conductivity with zero or one flux quanta per unit cell. The same effect can be observed at the low end of the next band. At higher energies, enhanced structure can be seen for $q > 1$, but the overall band conductivity is dominated by the increasing magnetic field. The positions of the Landau levels that would be observed in the absence of a periodic potential are marked on the figure for $p/q = 1$, and show that for a magnetic field of 0.4 T we are starting to resolve Landau-level-like structure in the band conductivity at these energies.

4.2. High field commensurability effects

For large magnetic fields ($\hbar\omega_c \gg V_0$) with p/q flux quanta to each unit cell of the potential, the individual Landau levels are expected to be split into p sublevels [22]. Figur3 shows for example the band conductivity for $p/q = 7/2$.

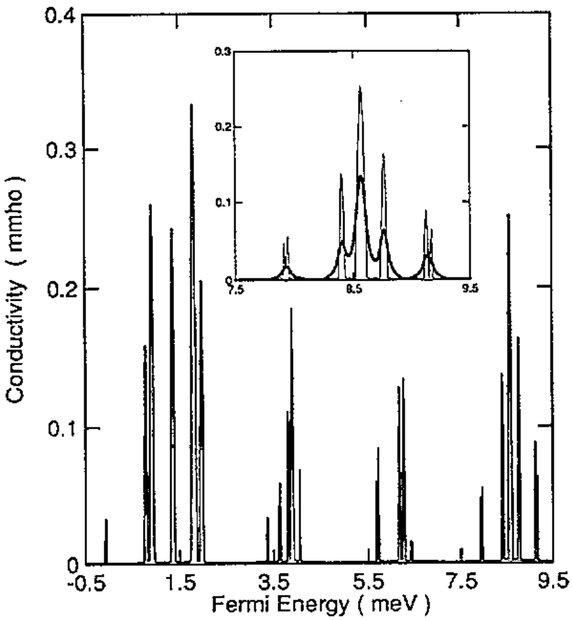

Figure 3. Band conductivity showing splitting of the lowest Landau level for $B = 1.4$ T, $p/q = \frac{7}{2}$. The inset shows in more detail the splitting of one of the Landau levels, calculated without any smoothing, and (bold line) smoothed by a Lorentzian corresponding to an inelastic scattering time of 10 ps.

The splitting in the lowest Landau level can be simply understood in terms of a local model. Figur4 shows the local density of states for the peaks in the density of states in the lowest Landau level with six flux quanta per unit cell. The states at low energies within the Landau level are seen to be edge states in the minimum of the potential, and the states at high energies are seen to be edge states around the maximum of the potential. Except around the centre of the split Landau level, the states have small weight passing through the saddle points from one unit cell to the next. The conductivity is therefore largest in the centre of the levels, as can be observed in figure 3. Since most of the states are confined around the minimum or maximum of the potential, they are only weakly broadened, and possibly split, by the periodic potential, and the conductivity is reduced. This splitting is the splitting considered by Gerhardts *et al* [6].

Since in a completely filled Landau level there is one electron of each spin per flux quantum, for a magnetic field with p flux quanta per unit cell, it is natural for the Landau levels to be split into p states.

Assuming that the states within a Landau level can be described as edge states circulating about either the minima or maxima of the potential, we can estimate the energies of the sub-levels, and the width of the split Landau level (see appendix for details). For large magnetic fields the position of the states is described well by the model; for smaller magnetic fields the cyclotron radius is larger, and the states are influenced by the non-parabolic nature of the potential. For all magnetic fields, the states nearer the centre of the level are closer together because of the non-parabolic nature of the potential.

In practice experiments are often performed by varying the magnetic field. We therefore show in figure 5 the behaviour of the density of states as a function of the magnetic field with a fixed Fermi energy, together with the predicted positions of the first and last split states of the lowest Landau level calculated by approximating the minimum and maximum of the potential by parabolic potentials. At low magnetic fields the fine structure is not resolved since the calculation is limited to magnetic fields with numbers p/q of flux quanta per unit cell, with $q < 7$. At larger magnetic fields Shubnikov–de Hass behaviour is seen,

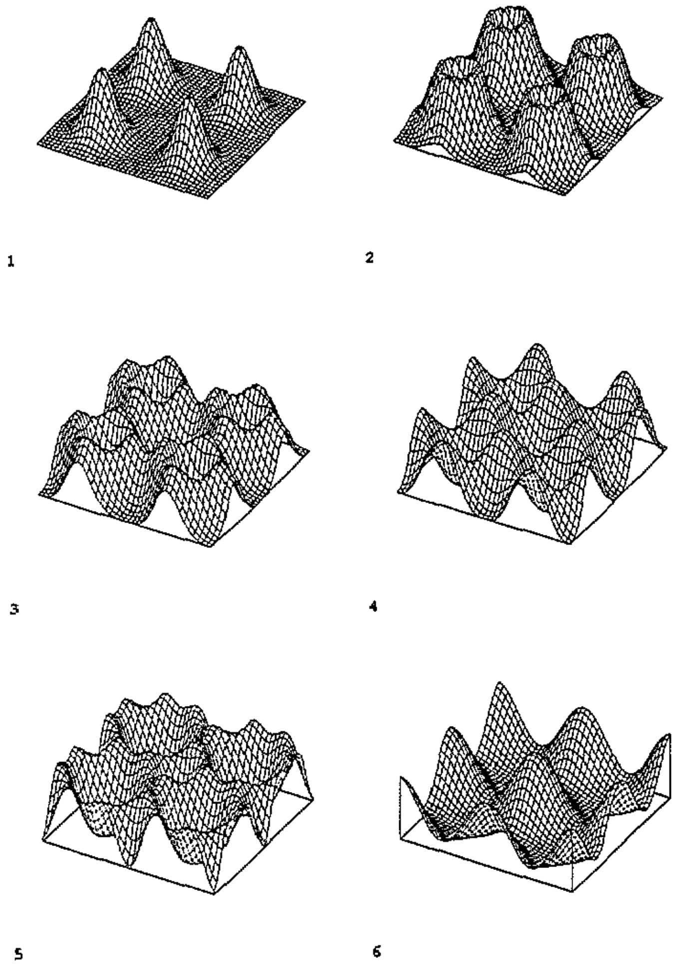

Figure 4. Local density of states for six flux quanta per unit cell. Energies are at the maxima of the density of states within the (split) first Landau level.

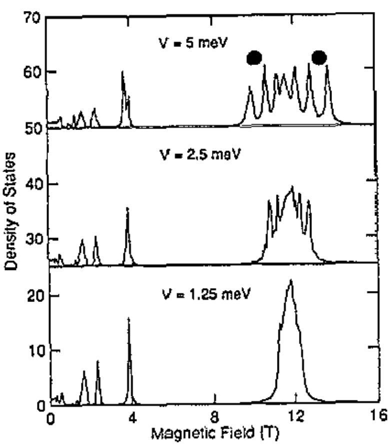

Figure 5. Density of states as a function of magnetic field at a Fermi energy of 10 meV and with different potential amplitudes V_0. The lattice period was 50 nm. The circles show the limits of the lowest Landau level calculated from the simple model in section 4.

with the peaks in the density of states broadened by the periodic potential, and the gross splitting of the Landau levels into edge states within the unit cells can be seen.

The model of states in a parabolic well breaks down for higher Landau levels, for the range of values considered here, because the predicted energies of the split states all lie in the region where the potential is not described well by a single parabola.

Since the mechanism described is local to one unit cell, we expect the gross splitting, and the consequent reduction in the conductivity, to be relatively easily observable in the presence of other physical mechanisms such as electron interactions and disorder. A simple estimate of the magnitude of these effects is obtained from the inelastic scattering time, which is of the order of 10 ps in zero magnetic field [40]. (The increased density of states in the presence of a magnetic field will tend to reduce this time.) Figure 3 shows that the effect of smoothing the conductivity by a Lorentzian corresponding to $\tau = 10$ ps is to hide the full Hofstader-like splitting, except in the central part of the Landau level. (For $p/q = \frac{7}{2}$ we expect between three and four sub-levels in the local picture.) For a shorter scattering time only the gross splitting into three sublevels will be visible.

5. Summary

We have presented results for the density of states and the band conductivity of a 2DEG in a periodic potential and magnetic field. We have shown how the method reproduces the cyclotron radius commensurability oscillations, and the scales on which the magnetic length commensurability effects are visible.

At low magnetic fields the band structure is complex, and except for the lowest, narrow, tight-binding band we do not see a clean splitting of the bands into the Hofstader butterfly. We do though see a reduction in the band conductivity, and added structure from band splitting, for non-integral numbers of flux quanta per unit cell.

For high magnetic fields, because the magnetic field suppresses the conductivity, the bands and Landau levels are narrow, and Hofstader's-butterfly like effects will be difficult to observe. The gross splitting of the Landau levels into sub-levels is a local consequence of the formation of edge states, and so is likely to be relatively robust.

Appendix

Expanding the potential V around the minimum

$$\partial^2 V/\partial x^2 = \tfrac{1}{4} V_0 \, (2\pi/a)^2 \tag{4}$$

gives a harmonic oscillator frequency ω_0 with

$$\omega_0^2 = \left(V_0/m_e a^2\right)\pi^2. \tag{5}$$

Solving Schrödinger's equation for the states in a parabolic well and magnetic field, one finds states at energies

$$\tfrac{1}{2}\hbar \left(\omega_c^2 + 4\omega_0^2\right)^{1/2} (2n + 1 + |m|) + \tfrac{1}{2}\omega_c \hbar m \tag{6}$$

where n, m are integers. For $\omega_0 \ll \omega_c$, the states that would be degenerate in the absence of the parabolic potential, have a separation

$$\Delta = \tfrac{1}{2}\hbar \left(\sqrt{\omega_c^2 + 4\omega_0^2} - \omega_c\right) \tag{7}$$

which is approximately $\hbar\omega_0(\omega_0/\omega_c)$, independent of Landau level. The first state in the Nth Landau level is at $(2N - 1)\hbar\omega_c/2 + N\Delta - V_0/2$. The same expansion can be performed about the maximum giving the highest state of the split band at $(2N-1)\hbar\omega_c/2 - N\Delta + V_0/2$.

For large magnetic fields, Δ is small, and the width of the split band approaches the width of the potential. For higher Landau levels the predicted energies of the first and last states cross, and the model must break down before this point due to the non-parabolic nature of the periodic potential.

References

[1] Ma J, Pucchiner R A, Liu W-P, Kriman A M, Maracas G N and Ferry D K 1990 *Surf. Sci.* **229** 341
[2] Weiss D, Grambow P, von Klitzing K, Menschig A and Weimann G 1991 *Appl. Phys. Lett.* **58** 2960
[3] Torimuri A, Ismail K, Burkhardt M, Antoniadis D A and Smith H I 1991 *Phys. Rev.* B **41** 12346
[4] Paris E, Ma J, Kriman A M, Ferry D K and Barbier E 1991 *J. Phys.: Condens. Matter* **3** 6605
[5] Liu C T, Tsui D C, Shayegan M, Ismail K, Antoniadis D A and Smith H I 1991 *Appl. Phys. Lett.* **58** 25
[6] Gerhardts R R, Weiss D and Wulf U 1991 *Phys. Rev.* B **43** 5192
[7] Degani M and Leburton J P 1991 *Phys. Rev.* B **44** 10901
[8] Fang H and Stiles P J 1990 *Phys. Rev.* B **41** 10171
[9] Weiss D, Roukes M L, Menschig A, Grambow P, von Klitzing K and Weimann G 1991 *Phys. Rev. Lett.* **66** 2790
[10] Ensslin K and Petroff P M 1990 *Phys. Rev.* B **41** 12307
[11] Beenakker C W J 1989 *Phys. Rev. Lett.* **62** 2020
[12] Weiss D, von Klitzing K, Ploog K and Weimann G 1989 *Europhys. Lett.* **8** 179
[13] Gerhardts R R, Weiss D and von Klitzing K 1989 *Phys. Rev. Lett.* **62** 1173
[14] Winkler R W, Kotthaus J P and Ploog K 1989 *Phys. Rev. Lett.* **62** 1177
[15] Beton P H, Alves E S, Eaves L, Dellow M W, Henini M, Hughes O H, Beaumont S P and Wilkinson C D W 1990 *Phys. Rev.* B **42** 9229
[16] Beton P H, Main P C, Davison M, Dellow M W, Taylor R P, Alves E S, Eaves L, Beaumont S P and Wilkinson C D W 1990 *Phys. Rev.* B **42** 9689
[17] Zhang C and Gerhardts R R 1990 *Phys. Rev.* B **41** 12850
[18] Gerhardts R R and Zhang C 1991 *Surf. Sci.* **229** 92
[19] Vasilopoulos P and Peeters F M 1989 *Phys. Rev. Lett.* **63** 2120
[20] Wharam D A, Thornton T J, Newbury R, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A and Jones G A 1988 *J. Phys. C: Solid State Phys.* **21** L209
[21] van Wees B J, van Houten H, Beenakker C W J, Williamson J G, Kouwenhouven L P, van der Marel D and Foxon C T 1988 *Phys. Rev. Lett.* **60** 848
[22] Claro F and Wannier G H 1970 *Phys. Rev.* B **19** 6068
[23] Thouless D J, Kohmoto M, Nightingale M P and den Nijs M 1982 *Phys. Rev. Lett.* **49** 405
[24] Wilkinson M 1987 *J. Phys. A: Math. Gen.* **20** 1761
[25] Hofstader D R 1976 *Phys. Rev.* B **14** 2239
[26] Kirczenow G 1992 *Surf. Sci.* **263** 330
[27] Kirczenow G 1992 *Phys. Rev.* B **46** 1439
[28] Oakeshott R B S and MacKinnon A 1993 *J. Phys.: Condens. Matter* **5** 6971
[29] Coleridge P T 1991 *Phys. Rev.* B **44** 3793
[30] Harrang J P, Higgins R T, Godall R K, Jay P R, Lamron M and Delescluse P 1985 *Phys. Rev.* B **32** 8126
[31] The calculation is performed as a superposition of one-dimensional calculations. With no scattering time put into the calculation the singularities in the one-dimensional densities of states obscure the two-dimensional density of states. All the calculations of densities of states therefore include the Lorentz broadening corresponding to a scattering time of 3.8 ps.
[32] Kelly M J 1985 *J. Phys. C: Solid State Phys.* **18** 6341
[33] Smoliner J, Rosskopf V, Berthold G, Gornik E, Böhm G and Weimann G 1992 *Phys. Rev.* B **45** 1915
[34] Středa P and MacDonald A H 1990 *Phys. Rev.* B **41** 11892
[35] Pippard A B 1962 *Proc. R. Soc.* A **270** 1
[36] Blount E 1962 *Phys. Rev.* **126** 1636
[37] Reitz J R 1964 *J. Phys. Chem. Solids* **25** 53
[38] Wulf U, Gudmundsson V and Gerhardts R R 1988 *Phys. Rev.* B **38** 4218
[39] Gudmundsson V and Gerhardts R R 1987 *Phys. Rev.* B **35** 8005
[40] Fasol G 1991 *Appl. Phys. Lett.* **59** 2430